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NOVEL PARALLEL ARCHITECTURES AND ALGORITHMS FOR LINEAR ALGEBRA PROCESSING

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ABSTRACT

Advanced problems in computational fluid dynamics, finite element structural analysis, and related areas require the solution of large partial differential equations and matrices of large size and dynamic range. This project considers an advanced parallel linear algebra processor and associated novel parallel algorithms for such applications. Research on system fabrication, quantitative performance evaluation and new parallel algorithms are described and considered. Case studies in structural mechanics, dynamics and nonlinear systems, finite element methods, computational fluid dynamics, and partial differential equations are included. The novel utilization of an optical processor for the processing of such problems is the major research given attention.

OVERVIEW

A brief overview of our project follows.

Our first year report (April 1986) detailed the linear algebra architectures being considered, the general nature of the partial differential equations to be solved, new number representation work, and initial laboratory system fabrication remarks. The goal of this project is to show the feasibility of a novel parallel matrix-vector linear algebra processor using optical processing techniques for the solution of partial differential equations for Finite Element Methods (FEMs) and Computational Fluid Dynamics (CFDs) applications. This work involves: architectural studies, novel number representation concepts, and parallel algorithms. It also involves modeling, simulation, error source analysis and laboratory system demonstrations. An optical

processor is considered to achieve these goals in a quite novel approach to parallel array processing.

YEAR 2 TASKS

The proposed Year 2 tasks and a brief discussion of each are given in Table 1. These tasks were enumerated more fully in our December 1985 proposal.

PROGRESS IN THE FIRST 6 MONTHS OF YEAR 2 EFFORT (APRIL 1986 - SEPTEMBER 1986)

Progress in the first 6 months of our second year of effort has been most excellent. Our emphasis has been on laboratory system data as we noted in our proposal. This has been achieved, despite the lack of equipment in our Year 2 NASA funding. The various aspects of this Year 2 effort during the first 6 months are now noted and summarized.

System Fabrication and Testing

We have fabricated and reported on a frequency-multiplexed optical linear algebra processor [1]. We have demonstrated its use in the laboratory for the solution of a parabolic partial differential equation (the transient diffusion equation in two spatial dimensions plus a time dimension [2]), and in the iterative solution of a system of linear algebraic equations [3] for the solution of the nonlinear matrix algebraic Ricatti equation [4]. We have fabricated a reduced one processor channel input modulator version of the multi-channel system that we intend to use. The present laboratory system employs one input acousto optic point modulator and 3 of the 10 encoding acousto optic channels in the multi-channel cell in the second plane of this system. The electronic support system for this processor is most extensive and of quite general purpose use. The system was tested, its performance quantified and its operation demonstrated [5] in the solution of a finite element plate bending problem (detailed in our April 1986 report). References 2 and 5 will be completed and transmitted to NASA by the end of October. All other references have been supplied to NASA earlier.

TABLE 1:
Proposed Year 2 Tasks

TASK NO.	TASK DESCRIPTION	TASK ITEM DISCUSSION
1	Laboratory system one-channel demonstration	Quantify error sources, nature, and performance
2	Basic algorithms	One-channel LU laboratory demonstration, 2 iterative algorithms and laboratory demonstration, low/high accuracy analog/digital algorithm and laboratory demonstration
3	Modeling	Match modeling and simulation to laboratory data for the algorithms in Task 2, select the best algorithm for a given architecture, system errors, data flow, partitioning, etc.
4	Simulation	
5	Data representation	Use the system tests and the results of Tasks 1-4 to quantify the data representation to be used in the different systems and architectures
6	Advanced algorithms	Nonlinear and time-varying algorithms for FEM and CFD case studies. This includes multi-mesh, level-slicing, and pivoting algorithms
7	Multi-channel architecture	Assemble this system and demonstrate it depending upon research progress in items 1-6
8	Advanced architecture	Other advanced architectures for specific algorithms, data flow, frequency-multiplexing, error source, and light budget considerations are possible and will be pursued as required.

This work is all most impressive and contains many novel features as detailed below:

1. The first laboratory optical linear algebra processor used to solve a real engineering problem.

2. The first frequency-multiplexed optical linear algebra processor demonstration.

3. The first iterative optical linear algebra processor laboratory demonstration.

4. The first nonlinear matrix solution on an optical linear algebra processor.

5. The first multi-channel optical linear algebra processor demonstration.

6. The first non-base-2 high-accuracy optical linear algebra processor laboratory demonstration

7. The first optical finite element processor solution on an optical linear algebra processor with laboratory demonstration.

8. The first direct solution of a linear algebraic equation on an optical linear algebra processor.

9. The first demonstration of our new one-channel LU decomposition algorithm.

10. Performance agreement with 21-bit VAX calculations and the computations achieved on the optical system were obtained.

11. The first multi-processor optical linear algebra system concept was detailed.

We have thus completed much of the tasks 1-4 and task 7 in our proposed research.

Data Representation and Advanced Concepts

We have considered a polynomial processor and found it to be useful for division. A single processor can be used and iterated and cascade of processors can be used with pipeline data in an architecture for such a problem. The use of such a polynomial processor in the partial differential equation approximate solutions is a future research topic.

We plan to use sign magnitude representation for the case of M = 1 input channel systems/algorithms and a negative base algorithm and sign number representation for the case when the number of input channels M is greater than 1. We plan to achieve floating point optical processing accuracy on the multi-channel system by optically processing the mantissa

data and digitally processing the exponential data, with the appropriate shifts of the input data performed in a digital preprocessor. The finite element problems we plan to consider will (hopefully) not require floating point accuracy. We will optically process all mantissa data by using the same exponent and by shifting the mantissa accordingly (or by analogous methods). Larger finite element and computational fluid dynamics problems will require floating point data. These problems will be achieved optically by the same technique (i.e. shifting the mantissa to provide normalized exponentials or by similar techniques). Thus, our optical treatment of data is identical to how floating point data will be handled on the full optical hybrid system. Our search for an even better floating point algorithm will continue through the end of our Year 2 research.

Algorithms

We have developed and demonstrated several new partitioning algorithms. We have demonstrated the use of our optical system to process data to variable accuracy by partitioning using a new algorithm. This allows us to trade accuracy for processing speed. We have described a new multiple optical processor architecture to realize this. The multiple-mesh algorithms we consider are related to our optical high/low accuracy algorithm. This applies mainly to finite element problems and is an established technique used in our new concept that is motivated by accuracy rather than by problem size and by calculation times. Our new partitioning technique is analogous to a new optical level-slicing algorithm that we proposed in our earlier 1986 proposed research. Several new treatments of the P_3 output (split CCDs and A/Ds and feedback of the binary encoded output per digit) are other advanced algorithm refinements we have recently addressed.

We have completed definition of our case studies and the algorithms to be considered for each finite element and computational fluid dynamics problem. The computational fluid dynamics problem to be addressed and our case study concerning it will involve a driven cavity.

The finite element solution to this problem will involve optical solution of the steady state Navier-Stokes equation. This represents a nonlinear time-independent equation solution. This solution will be obtained by Newton-Rhapson techniques involving new Jacobian update methods. We intend to compare the results of iterative and direct solutions for this method. Since the Jacobian is of small size, direct techniques should result in nearly the same number of calculations and computations as result in or for the iterative solution. These issues will be quantified in our numerical studies. Initial results are expected in the next 6 months of research. In the finite difference solution of the same driven cavity computational fluid dynamics problem, we will address a time-dependent Navier-Stokes solution. This involves a nonlinear time-varying equation solution. The flexible NASA program we have been provided with will be used in this problem. NASA studies found methods involving Successive Overrelaxation (SOR) and Alternating Direction Implicit (ADI) algorithms to be preferable solutions. We will investigate both of these solutions in our research. Two other solutions that appeared to be better from an optical standpoint are the hopscotch and BDM algorithms. These algorithms will also be addressed in our Year 3 proposed research. We anticipate initial results on these various algorithm implementation details to be included in our 1986 final Year 2 report. Our finite element problem definition was formulated and detailed earlier. It includes a dynamic plane-frame finite element structural problem using earthquake loads. It involves a forced non-steady state response (not a transient solution) to the partial differential equations involved. We will invoke and utilize the Newmark algorithm as well as modal analysis solutions for the eigenvalues and eigenvectors. We note that these last modal analysis solutions are not necessarily attractive for optical realization (because of their extensive computational accuracy required). This issue will be addressed in our forthcoming research during the next 6 months. By the conclusion of our next 6 months of research, we anticipate that the algorithms to be addressed will have been finalized, and that initial utilization of our algorithm studies will have been finalized.

We have submitted a request for time on the new Cray XMP processor in Pittsburgh. We have recently been notified that this request has been approved. Thus, we do not anticipate requiring the use of NASA Langley computer time that has been offered. We do not anticipate or foresee the need for a meeting at NASA until December (by this time we plan to have initial algorithm simulation results). I will be in contact with NASA to arrange this December meeting.

FUTURE WORK PLANS (APRIL 1986 - MARCH 1987)

We will be completing two conference papers on the parabolic partial differential equation and finite element solution run on our laboratory system, as well as preparing journal versions of the finite element and the parabolic partial differential equation laboratory results.

TASK 1: We have obtained much detailed knowledge from our laboratory tests and demonstrations. There is significant temperature drift in the present laboratory system. We plan to overcome this with a new ac coupled laboratory system. We plan to report on initial fabrication efforts of this system in the final period of this Phase 2 effort. We will fabricate and use an initial laboratory version of this system in our Year 3 effort. The light budget of this system will be improved to allow higher dynamic range processing to be achieved by using ac coupling in the new system, ac modulated laser diode point modulators as the P_1 input transducers, and improved detectors at the output P_3 system. We plan to fabricate an initial one-channel version of this system. New detectors and amplifiers will be obtained to increase system dynamic range and to improve system drift and noise response. We hope to purchase these items from other equipment sources. We expect this improved system to allow higher radix data to be handled and more input channels to be allowed in the processing.

With encoded data in high-accuracy processors, no error source dominates an iterative algorithm. Dominant error sources exist in analog algorithms, where errors are perturbations to the analog values. With digital encoding, this is not the case as we have shown earlier [6]. Our

simulations indicate that large errors are possible with encoded data with the possibility of perfect accuracy being quite high. If the error sources increase and become too large, their effects can cause errors of any magnitude (by nature of the encoded data). We expect digitally-encoded algorithms to correct errors (in iterative algorithms), with the number of iterations required increasing proportional to the amount of error in the result (this can be large when encoded data is employed). Verification of this will occur in the next 6 months and in our case studies planned for Year 3 work. We plan to pursue new compact Crout and other algorithms which do not formulate direct solutions as matrix-matrix multiplications. These algorithms are expected to be preferable to the LU algorithm when a multi-channel system is available (the LU algorithm does not use the multi-channel system to its best efficiency, i.e. it only employs one channel of such a system). The status of this new algorithm is expected to be available in our April 1987 final report.

TASK 2: We have successfully demonstrated our one-channel LU decomposition algorithm. In the next 6 months, we plan to demonstrate our high/low accuracy iterative algorithm (on the analog frequency-multiplexed processor). The analysis and operation of such a two accuracy level system is becoming more complex than initially anticipated. We expect all iterative algorithms to perform similarly. If multiple right hand sides exist in the linear algebraic equations, then direct algorithms are preferable. If a good guess at the answer exists, we expect iterative algorithms to perform better. This will be quantified in our Year 3 case studies. Our preferable Crout algorithm work will also be given initial attention.

TASK 3: Our error source modeling is complete (as far as presently anticipated). We plan to modify our simulator to perform faster for large matrix problems. This work and initial results on it will be detailed in our April 1987 final report.

TASK 4: Problems in this area are presently being investigated and will be reported upon in our final report and will be further detailed in our Year 3 final report.

TASK 5: We have designed a P_3 detector back-end capable of handling various number representations. This system is useful, since it demonstrates the flexibility possible on our proposed architecture. We will document this and include it as an appendix in our final report of April 1987. We will not fabricate this P_3 detector system, due to lack of equipment funding. We plan to use sign-magnitude representation for one-input channel systems and our LU decomposition algorithm. We will modify the back-end software (rather than fabricating hardware) to utilize negative base representation. We plan to employ this number representation for bipolar data with multi-channel input operational systems in our laboratory processor. New floating point techniques may emerge with future research studies. The software to handle negative base and floating point data will be provided in our future research efforts.

TASK 6: We will complete our study of pipelined polynomial architectures for approximate partial differential equation solutions. We do not anticipate optical perfect shuffle and pivoting algorithms to be of use in this effort. However, we will consider other on-line arithmetic algorithms further for use in various resolution and parallelism properties available and possible in optical systems. Our variable accuracy partitioning technique is one attractive version of level-slicing that we plan to employ in future work. Multi-mesh algorithms will also see use in our Year 3 case study work. The search for better floating point algorithms will continue. The Crout algorithm will be considered also.

TASK 7: Our initial system was fabricated, tested and demonstrated. The results from this system provided input on the improved system using ac coupling that we plan to fabricate. This ac system will increase the number of data levels in P_1 and P_2 input channels we can allow, depending upon available funding. Displays necessary for FEM and CFD data will be considered.

TASK 8: For improved throughput, we will multiplex the input data to 3 x 40 MHz = 120 Mbs and will demultiplex the output for input to the different memory channels available. The cost for this hardware and the new high-speed P_3 arithmetic logic units will hopefully be provided by other sources. Frequency-multiplexing will not be employed in the laboratory system fabricated. We have earlier demonstrated its use but will not employ it in the present laboratory system (to reduce hardware costs), since including frequency multiplexing will require multiple P_3 hardware. Frequency-multiplexing allows more than one vector inner product to be calculated in parallel. We will use the availability of this frequency-multiplexing concept in system block diagrams to increase the throughput of the system and as a technique to handle bipolar data [1] (although negative base appears to be preferable, since it requires only one output P_3 system). Our new ac coupled system, laser diode sources for each input channel (ac modulated), new output P_3 detectors, and better mounts should improve the performance possible and the results obtainable on our laboratory system.

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